2.3 Set Operations

Union of sets A and B.

A U B = { x : x is an element of A or
x is an element of B }

A = { 1, 3, 6, 7 }
B = { 2, 3, 5, 6 }

A U B =

Intersection of sets A and B.

A ∩ B = { x : x is an element of A and
x is an element of B }

A = { 1, 3, 6, 7 }
B = { 2, 3, 5, 6 }

A ∩ B =

Complement of set A.

A' = { x : x is <u>not</u> an element of A }

U = { 1, 2, 3, 4, 5, 6, 7, 8 } (Universe)
B = { 1, 3, 6, 7 }

A' =

Set difference of sets A and B.

B – A= { x : x is an element of B and
x is <u>not</u> an element of A }
"B take away A"

A = { 1, 3, 6, 7 }
B = { 2, 3, 5, 6 }

A – B =

Order of Operations

( ) parenthesis always done first.

' set complement next.

U, ∩, – union, intersection, difference
last

U = { 1, 2, 3, 4, 5, 6 }
A = { 1, 2, 3, 4 }
B = { 2, 4, 6 }

Find A U B'

U = { 1, 2, 3, 4, 5, 6 }
A = { 1, 2, 3, 4 }
B = { 2, 4, 6 }

Find (A ∩ B)'

U = { 1, 2, 3, 4, 5, 6 }
A = { 1, 2, 3, 4 }
B = { 2, 4, 6 }

Find A' – B

U = { 1, 2, 3, 4, 5, 6 }
A = { 1, 2, 3, 4 }
B = { 2, 4, 6 }

Find (A U B) ∩ B

U = { 1, 2, 3, 4, 5, 6 }
A = { 1, 2, 3, 4 }
B = { 2, 4, 6 }

Find A' – (A ∩ B)

Order of Set Operations

- Example: Let $U=\{1,2,3,\ldots,10\}, E=\{x : x \text{ is even}\}$, $B=\{1,3,4,5,8\}$, and $A=\{1,2,4,7,8\}$.

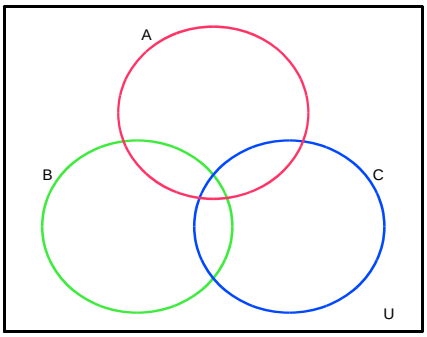
  Find $(A \cup B)' \cap (E' \cup A)$.

  1. $(A \cup B) = \{1, 2, 3, 4, 5, 7, 8\}$
  2. $(A \cup B)' = \{6, 9, 10\}$
  3. $E' = \{1, 3, 5, 7, 9\}$
  4. $(E' \cup A) = \{1, 2, 3, 4, 5, 7, 8, 9\}$
  5. $(A \cup B)' \cap (E' \cup A) = \{9\}$

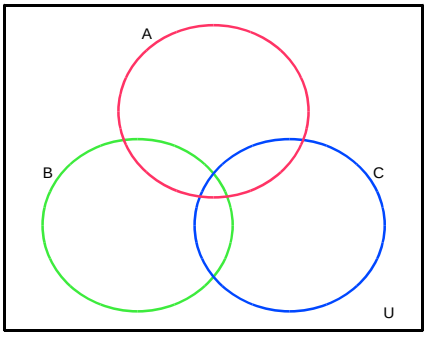
DEMORGAN'S LAWS FOR SET THEORY
If A and B are sets, then $(A \cup B)' = A' \cap B'$ and $(A \cap B)' = A' \cup B'$.

U = { 1, 2, 3, 4, 5, 6 }
A = { 1, 2, 3, 4 }
B = { 2, 4, 6 }
Verify that (A U B)' = A' U B' in this case.

Three sets can also be represented in a Venn Diagram.



U = { 1 … 10 } A = { 1, 2, 3, 5, 7 }
B = { x : x is even } C = { x : x is odd }



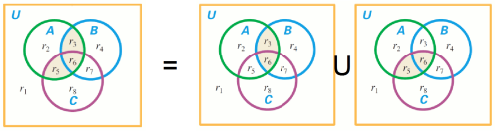
U = { 1, 2, 3, 4, 5, 6 }
A = { 1, 2, 3, 4 }
B = { 2, 4, 6 }
C = { 3, 4, 5 }

Find (A U B) – C'

Order of Set Operations

- Intersection distributes over union.

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$



$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

This same phenominon occurs in the distributive property in the integers.

R x (S + T) = (R x S) + (R x T)

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

Union also distributes over intersection.

Order of Set Operations

THE CARDINAL NUMBER OF THE UNION OF TWO SETS

If A and B are sets, then $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

We must subtract $n(A \cap B)$ so we do not count these elements twice.